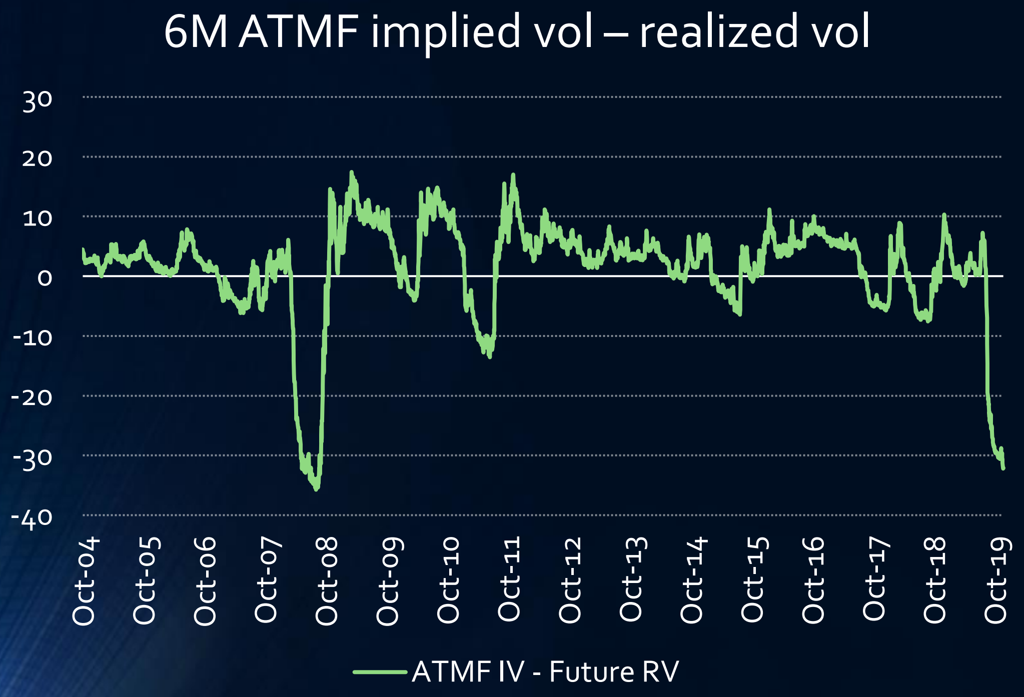
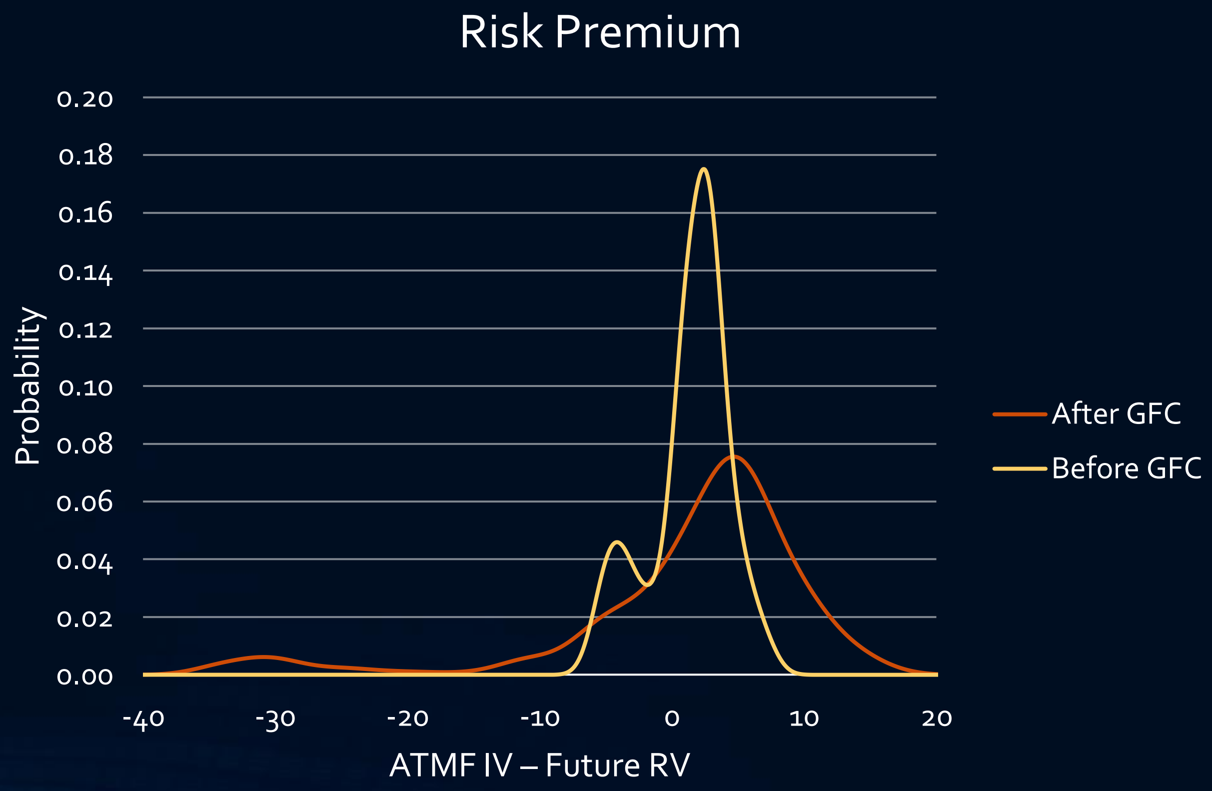
Locally Short Volatility & Globally Long Convexity

BACH OPTION

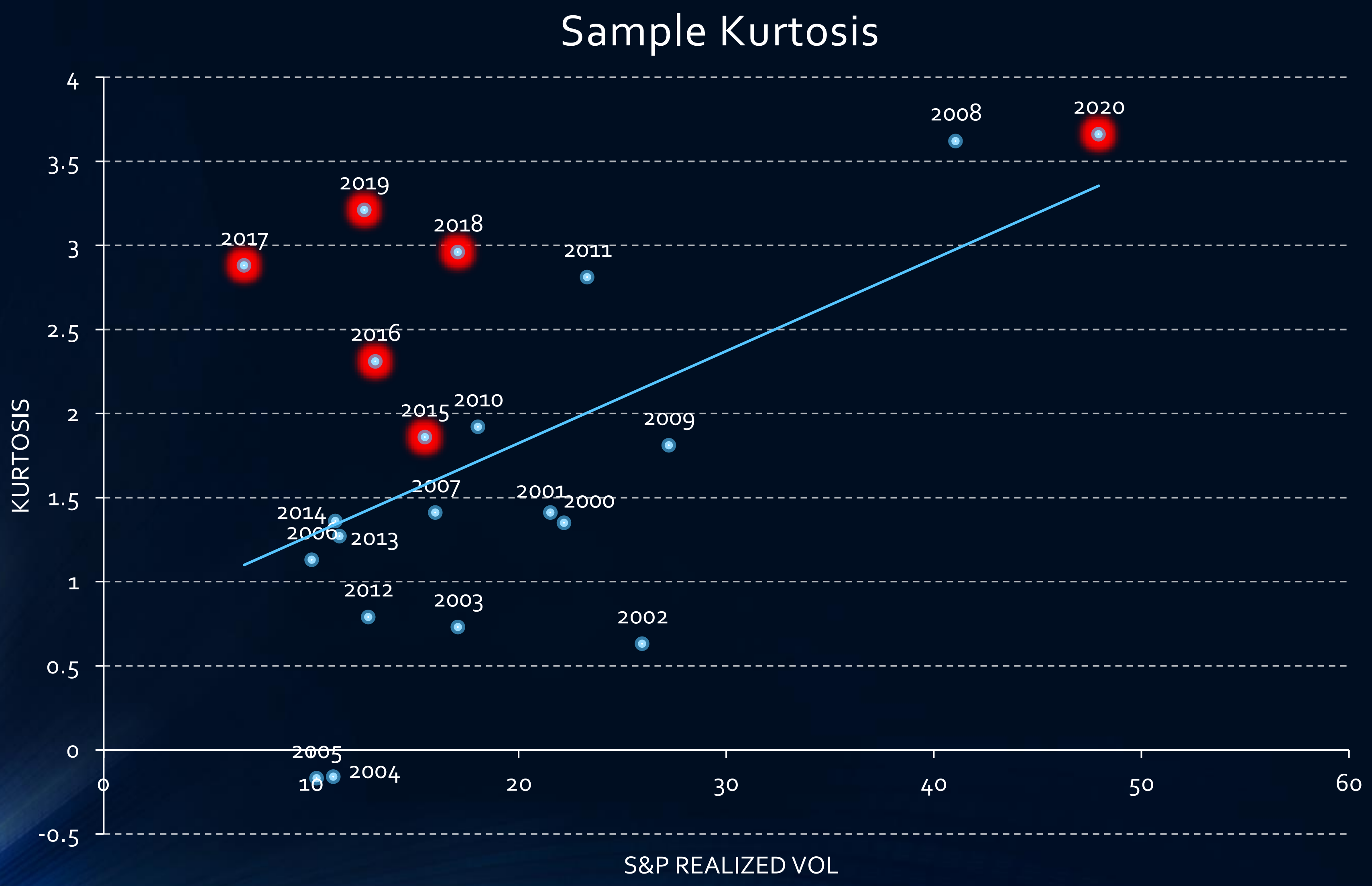
- Volatility risk premium associated with equity index options are quite high and therefore are worth exploiting.
- On the other hand, volatilities are so prone to infrequent and yet cataclysmic upward jumps, as seen in 2008, 2018 and this year that a short position can be ruinous.
- Without great foresight, how to resolve this dilemma?
- We present a simple framework within which one is short volatility locally , but long convexity globally.

SPX Volatility Risk Premium



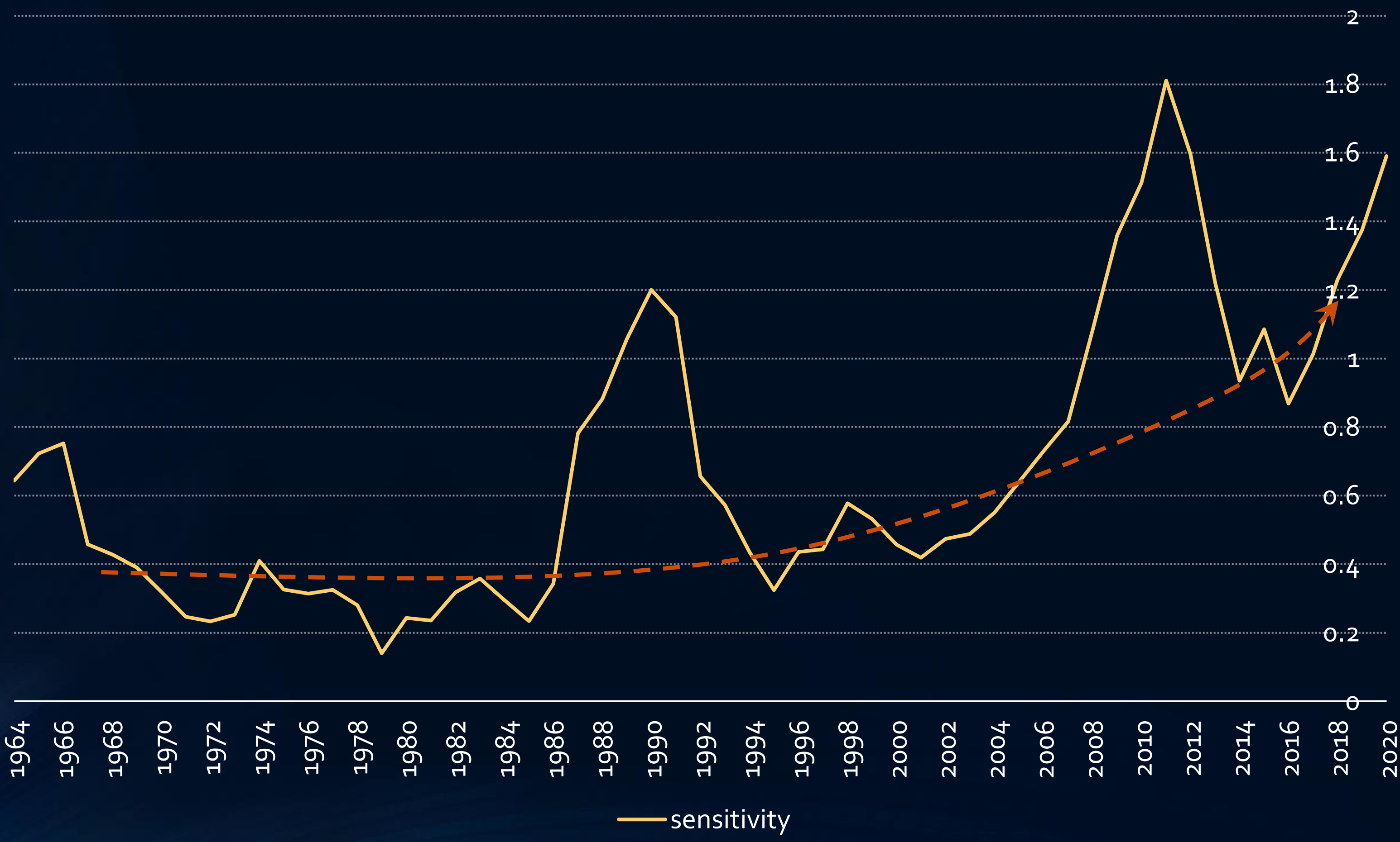


Kurtosis Increasing

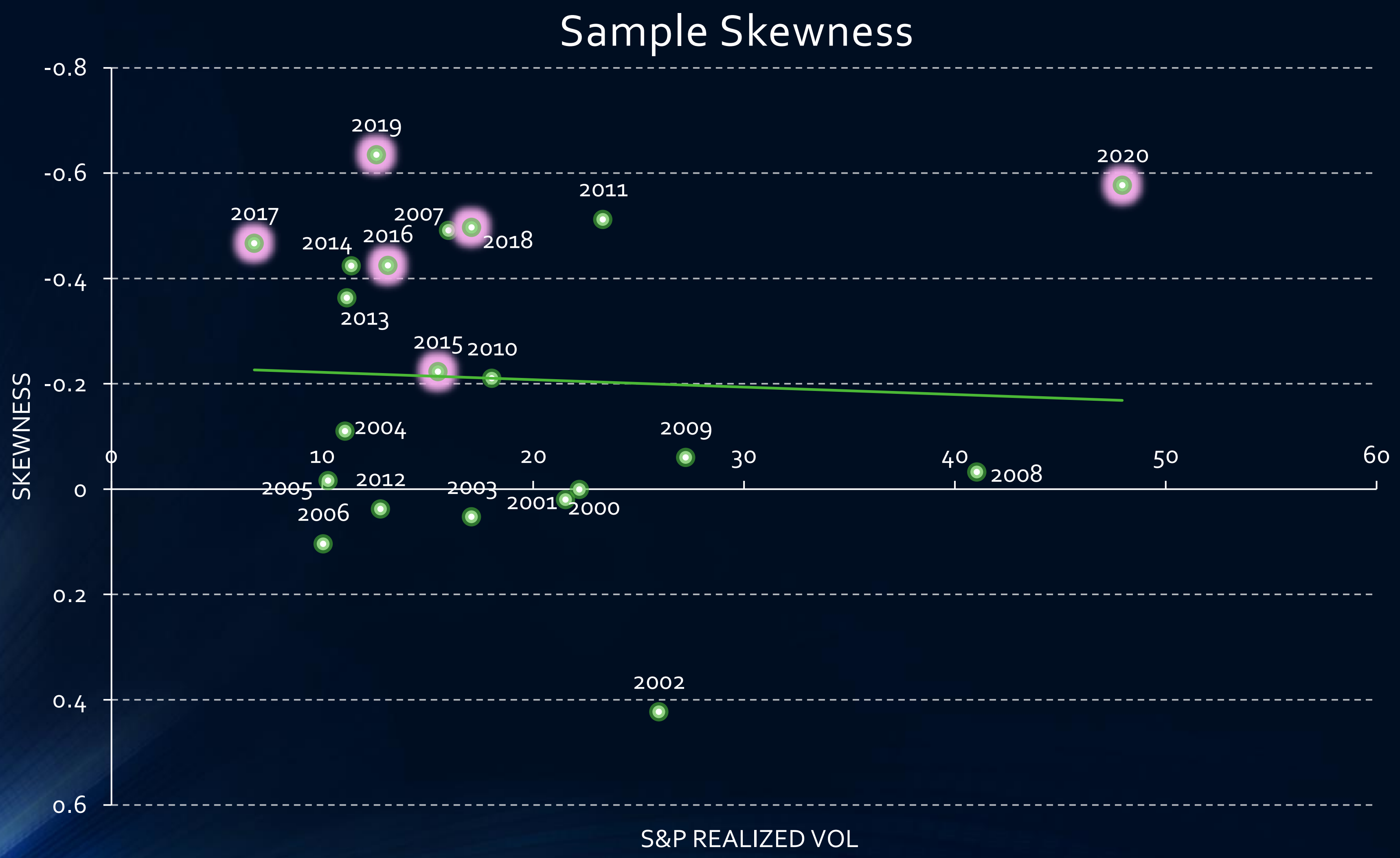


Increase of Realized Volatility per -1% Monthly Return

5-year rolling, SPX monthly data



Negative Skewness Increasing

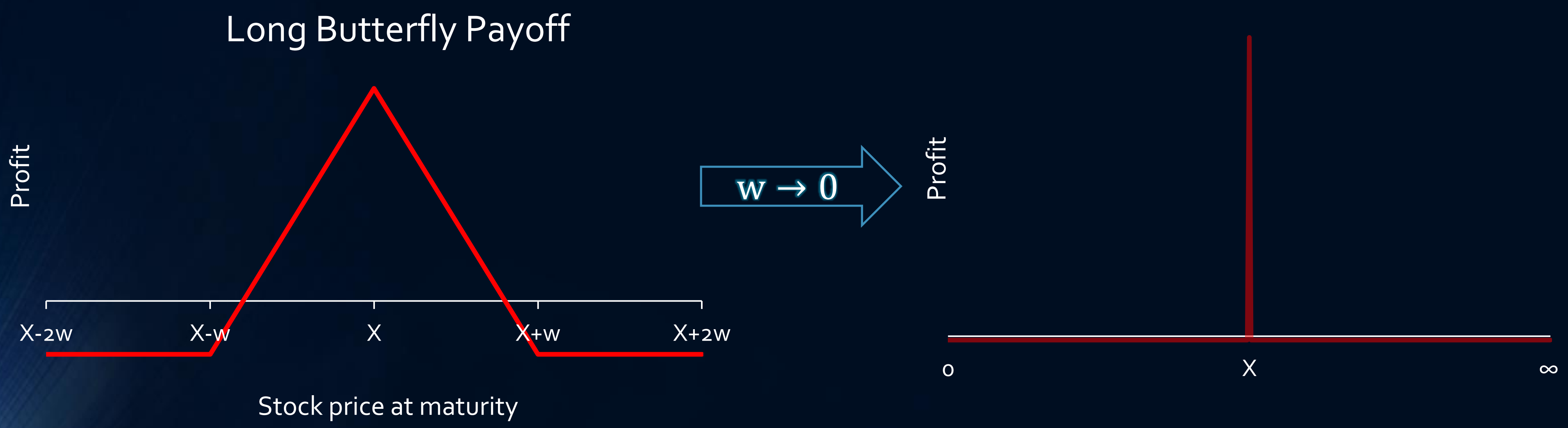


Why So Binary?

- High kurtosis reflects high vol of vol: $E[X^4] \sim (E[X^2])^2 + Var(E[X^2])$.
- The market alternates between the state of low-volatility upward drift and that of volatile downward rout.
- The fundamentals have been lackluster. With the exception of a subset such as FANG, sentiment towards risk assets has not been so euphoric as to drive up upside volatility.
- Fed & Stock Buyback mitigate downside.
- Value investing: buy low, sell high; Momentum investing: buy high, sell low.
- Prevalence of momentum investing exacerbate the inherent instability.

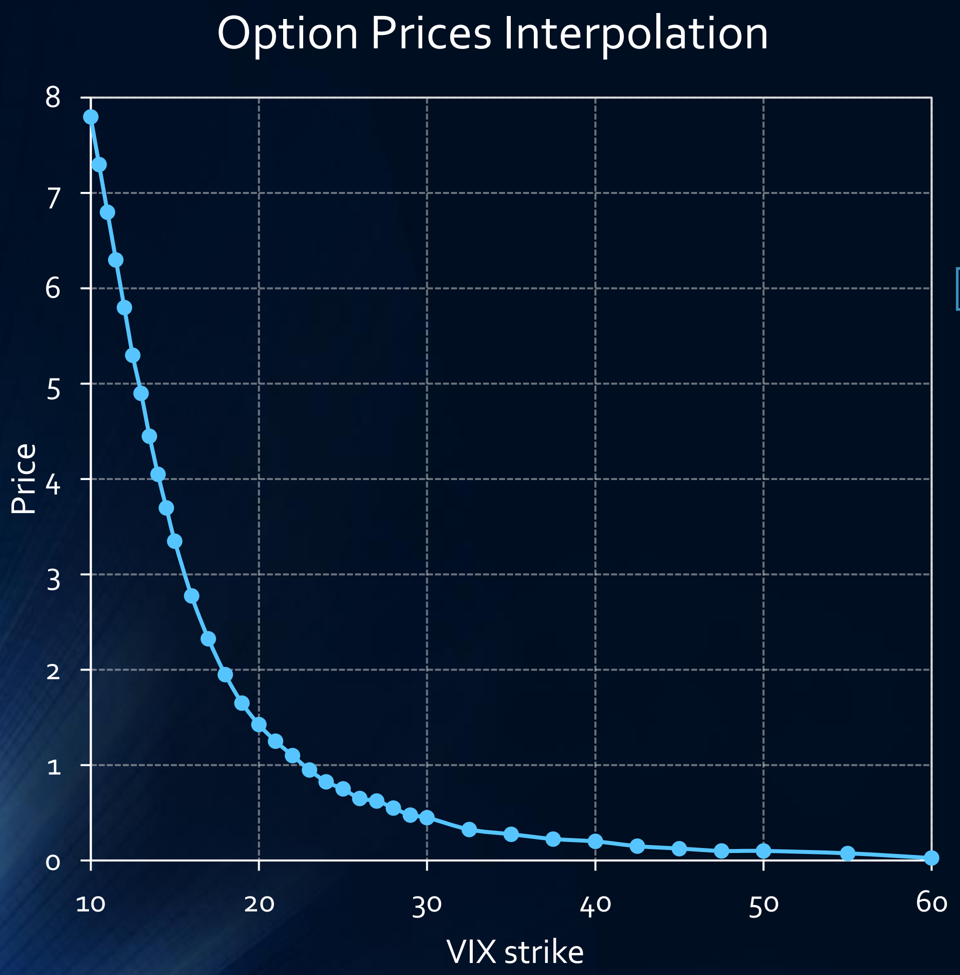
Probability distribution inferred from option prices

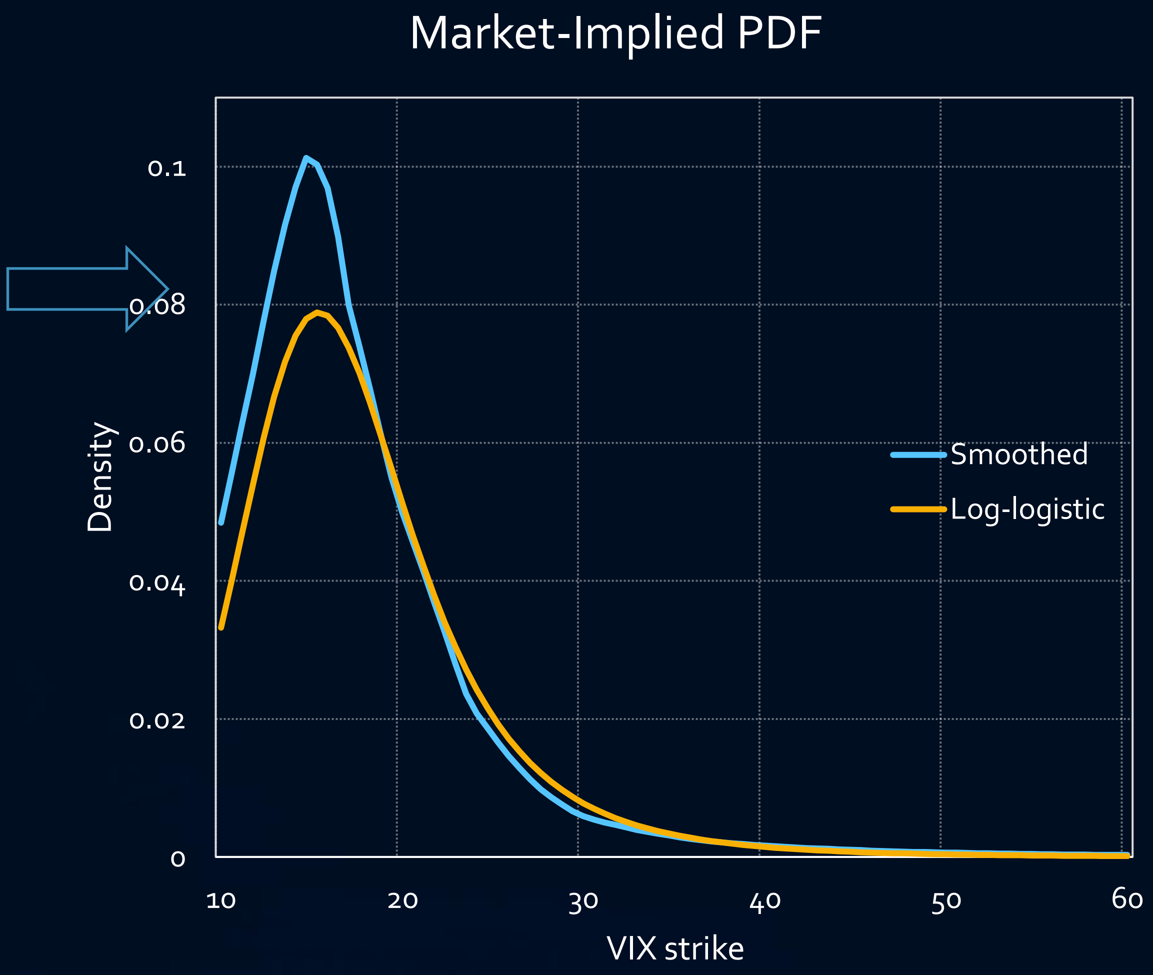
- Consider a long butterfly option position: long 1 call with strike X - w, short 2 calls with strike X, long 1 call with strike X + w, the payoff is the following :



- When w → 0, the payoff at X tends to the probability density for the stock price = X at maturity. We can impute the market-implied probability distribution for the stock price at maturity from option prices.

Probability distribution inferred from option prices



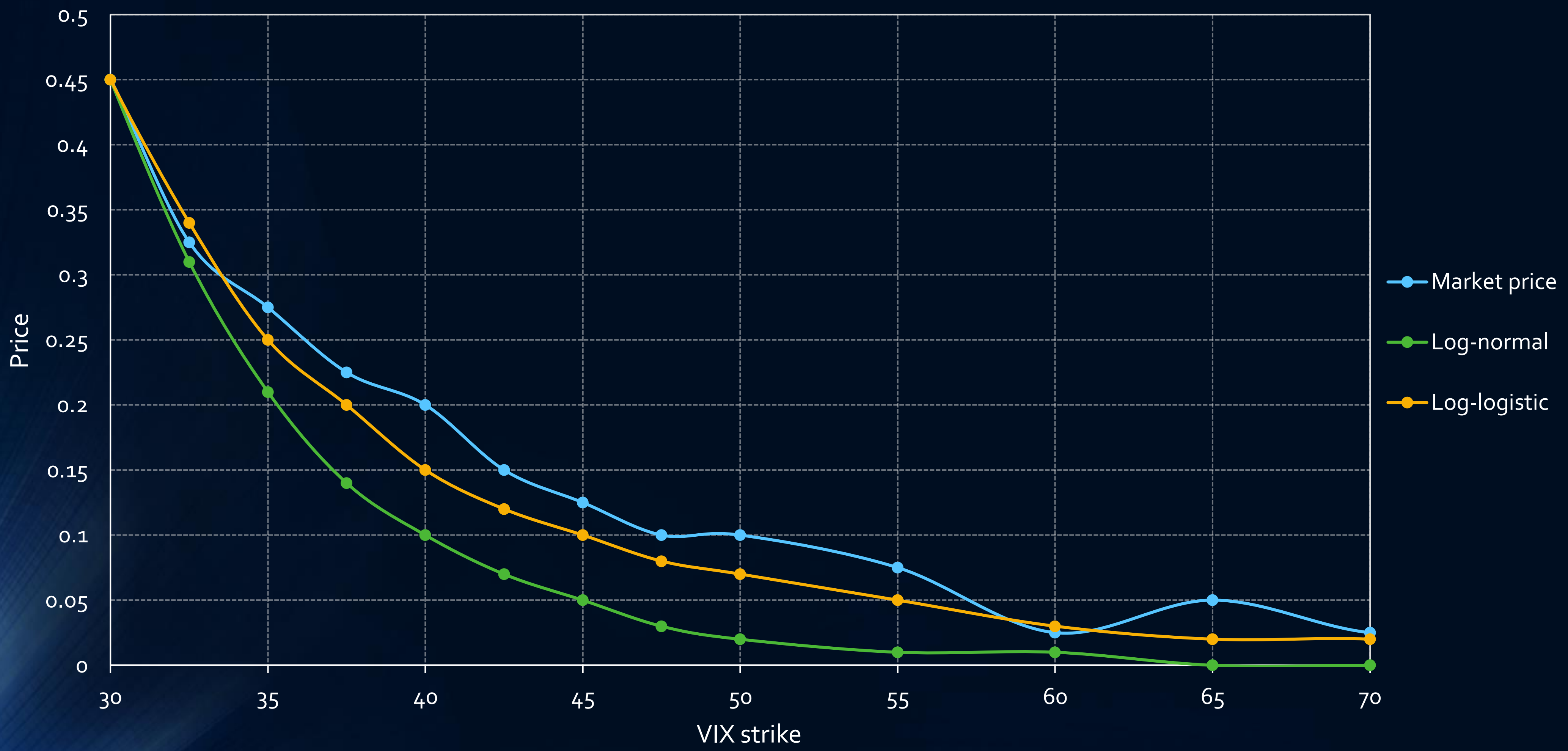


*Market prices taken from Bloomberg as of COB Jan 27, 2020, all options mature on Feb 19,2020.

VIX terminal probability

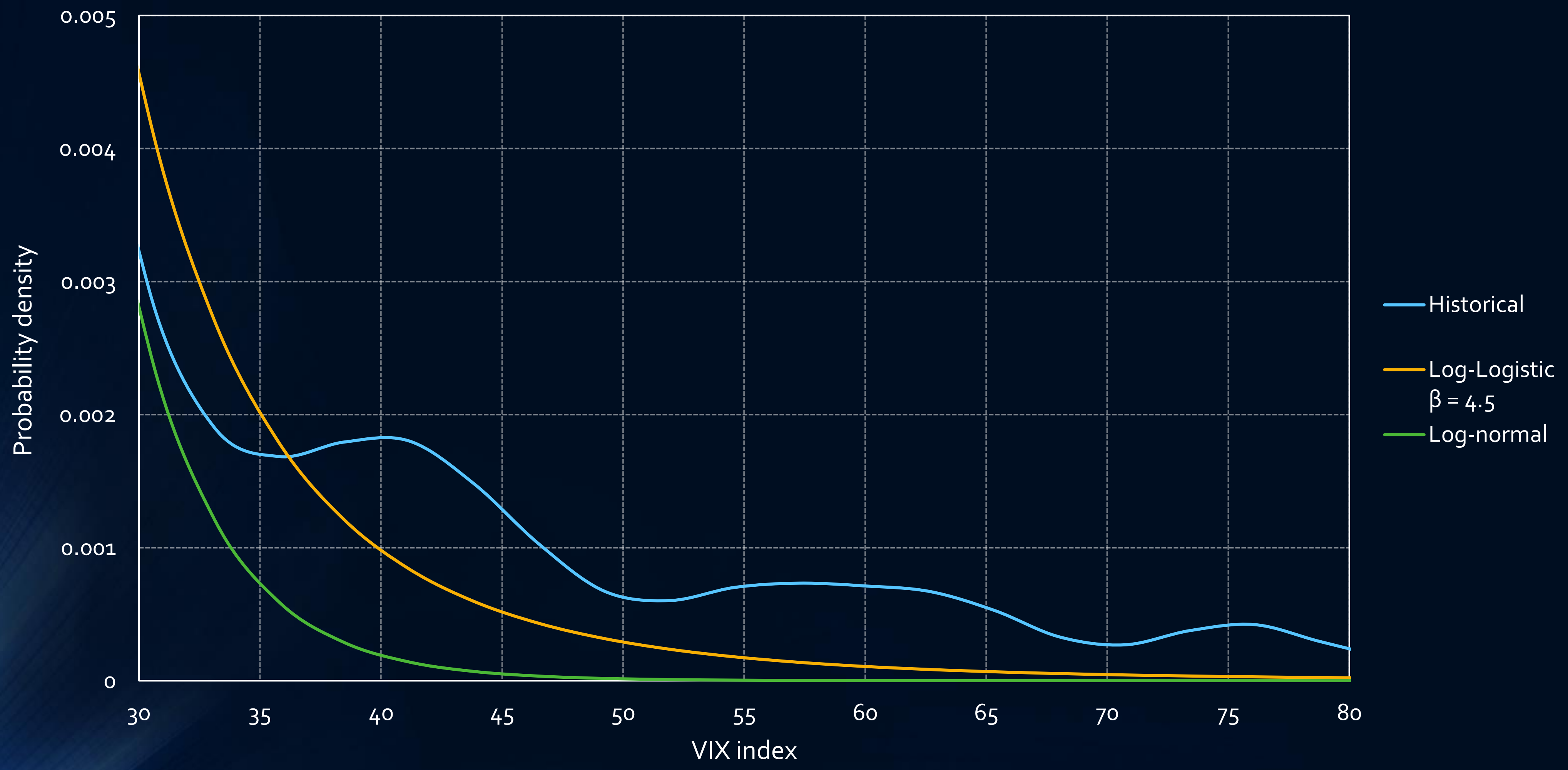
- Interpolation is required to obtain the market-implied PDF.
- Alternatively we can fit some parametric distribution to the option prices.
- A common choice is the log-normal distribution. It fits well for strikes near the current spot but tends to underestimate the magnitude of the tails.
- The distribution we consider as both heavy-tailed and analytically trackable is the Log-Logistic distribution. Its CDF $= \frac{x^{\beta}}{\alpha^{\beta}+x^{\beta}}$ where β determines the thickness of the tail.
- Asymptotically, P(x > Z) $\propto Z^{-\beta}$ when Z → ∞ .

Log-normal vs Log-logistic repricing difference



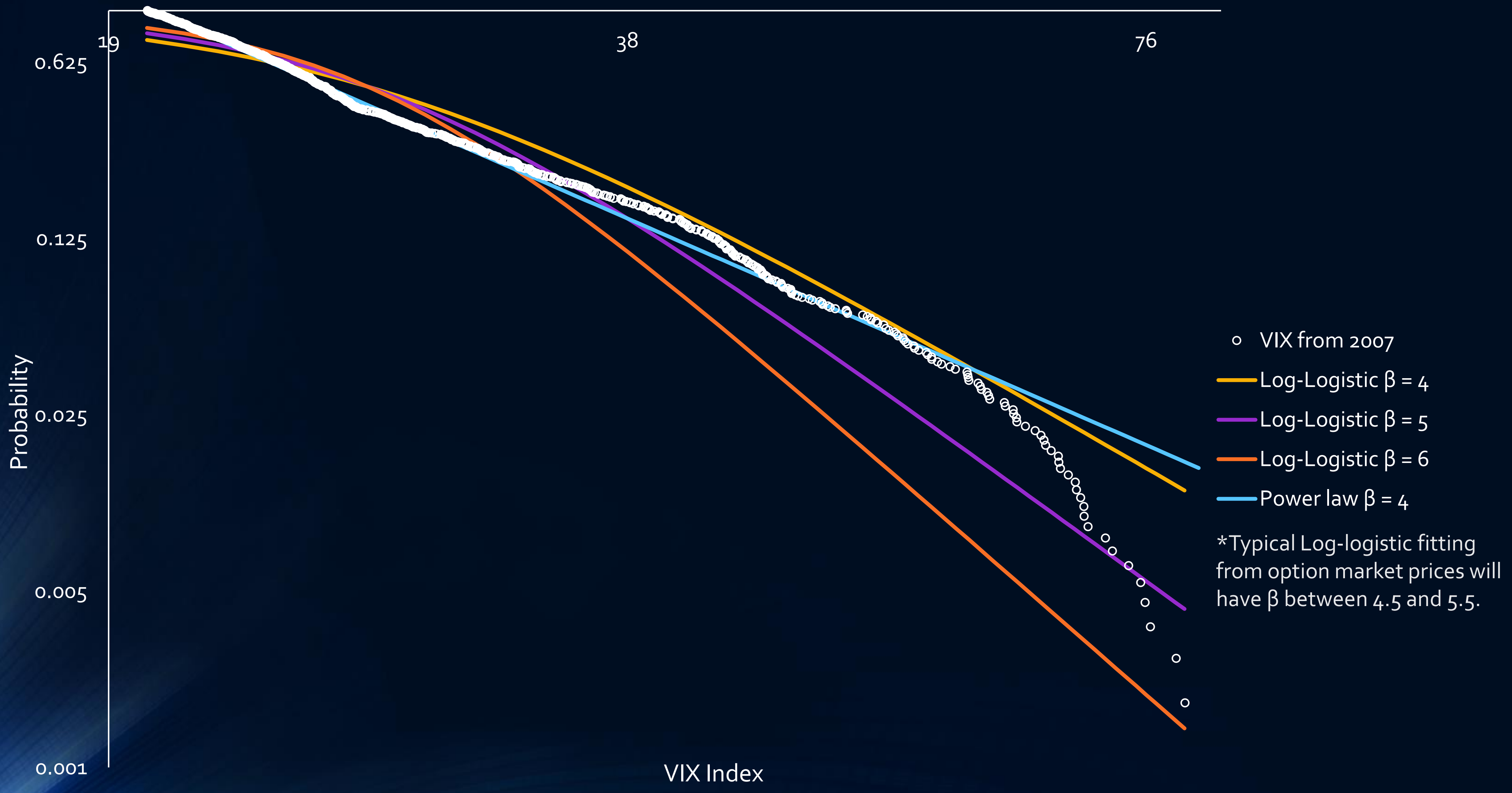
*Market prices taken from Bloomberg as of COB Jan 27, 2020, all options mature on Feb 19,2020.
*Parametric distributions are fitted to option prices such that 1) the expectation = VIX future 2) 30-strike call price = market price.
*Parameters fitted for two distributions: log-logistic: $\alpha = 16.5$, $\beta = 4.77$; log-normal: meanlog = ln(16.3) , sdlog = 0.41.

Historical vs market implied pdf tail difference



*Historical probability distribution is smoothed from daily VIX index level starting from year 2016.

Historical vs theoretical probability difference



Variance Swap

- Payoff $\propto (Realized\ vol)^2 - {\sigma_{var}}^2$

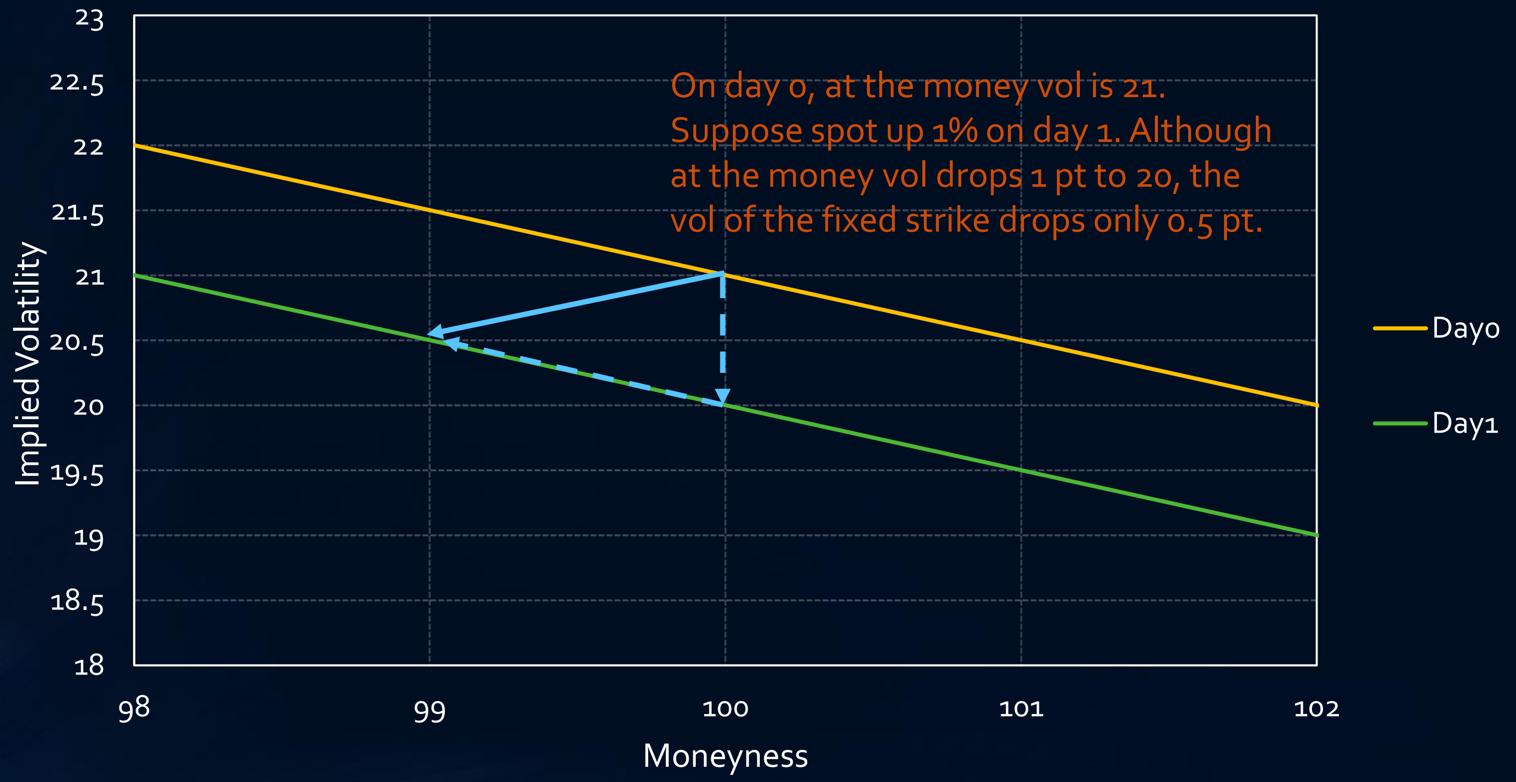
    where σ_{var} is the strike of the contract.

- Path-independent.
- Provides pure exposures to $volatility^2$.
- E.g. Suppose one is long of 1 vega variance swap when the strike is 20. If the realized volatility turns out to be 40, then the pnl would be 30 vega.

Vanilla Option

- $\Delta\, IV(K)_t \approx \Delta ATM_t + b \cdot \Delta S_t$

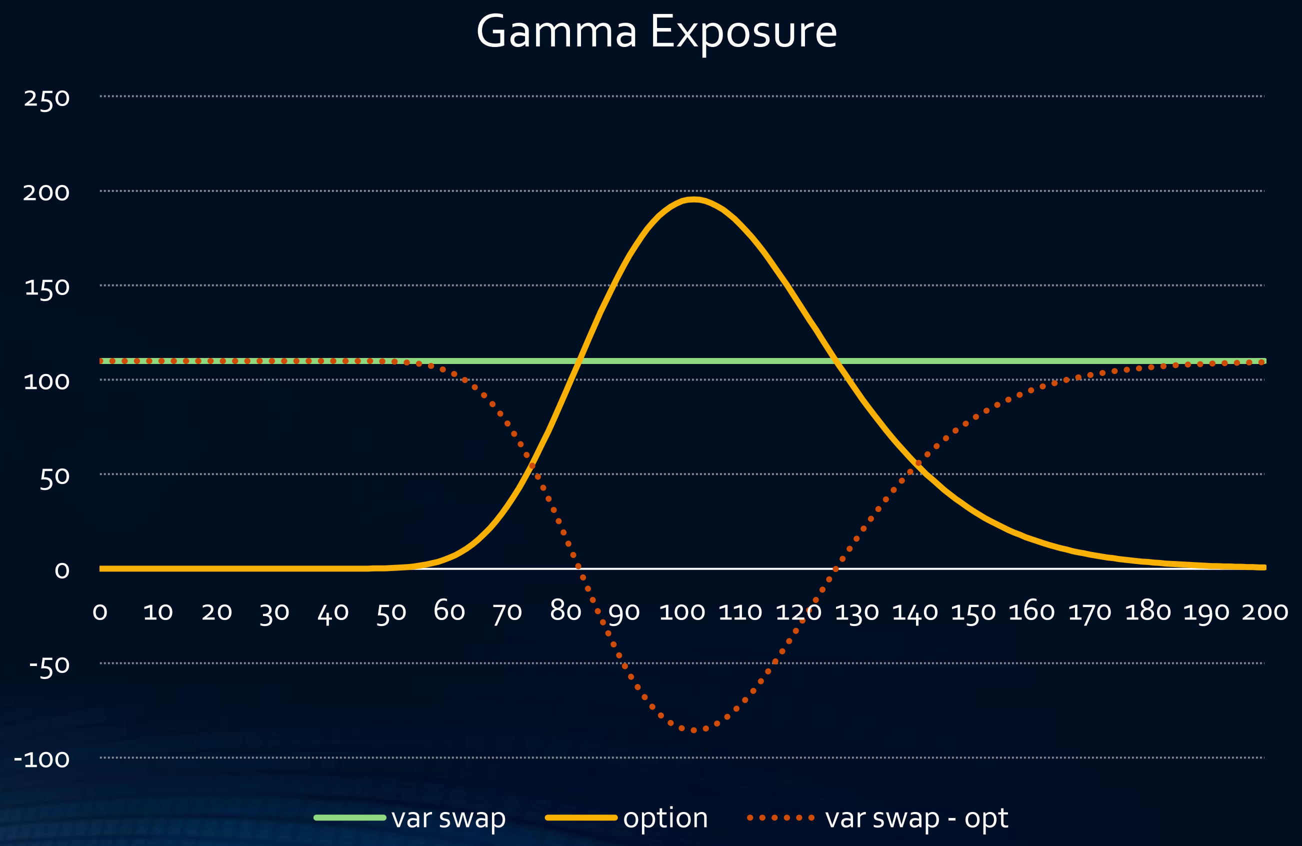
where K is the fixed strike, b denotes the slope of the implied volatility curve, S is the underlying spot price.



- If the change in the ATM implied vol is 0, the more the spot S increases, the higher the fixed strike vol.

Example Strategy

- ❖Variance swap: constant Gamma exposure
- ❖Vanilla option: Gamma reaches maximum near strike and decreases as spot moves further from strike

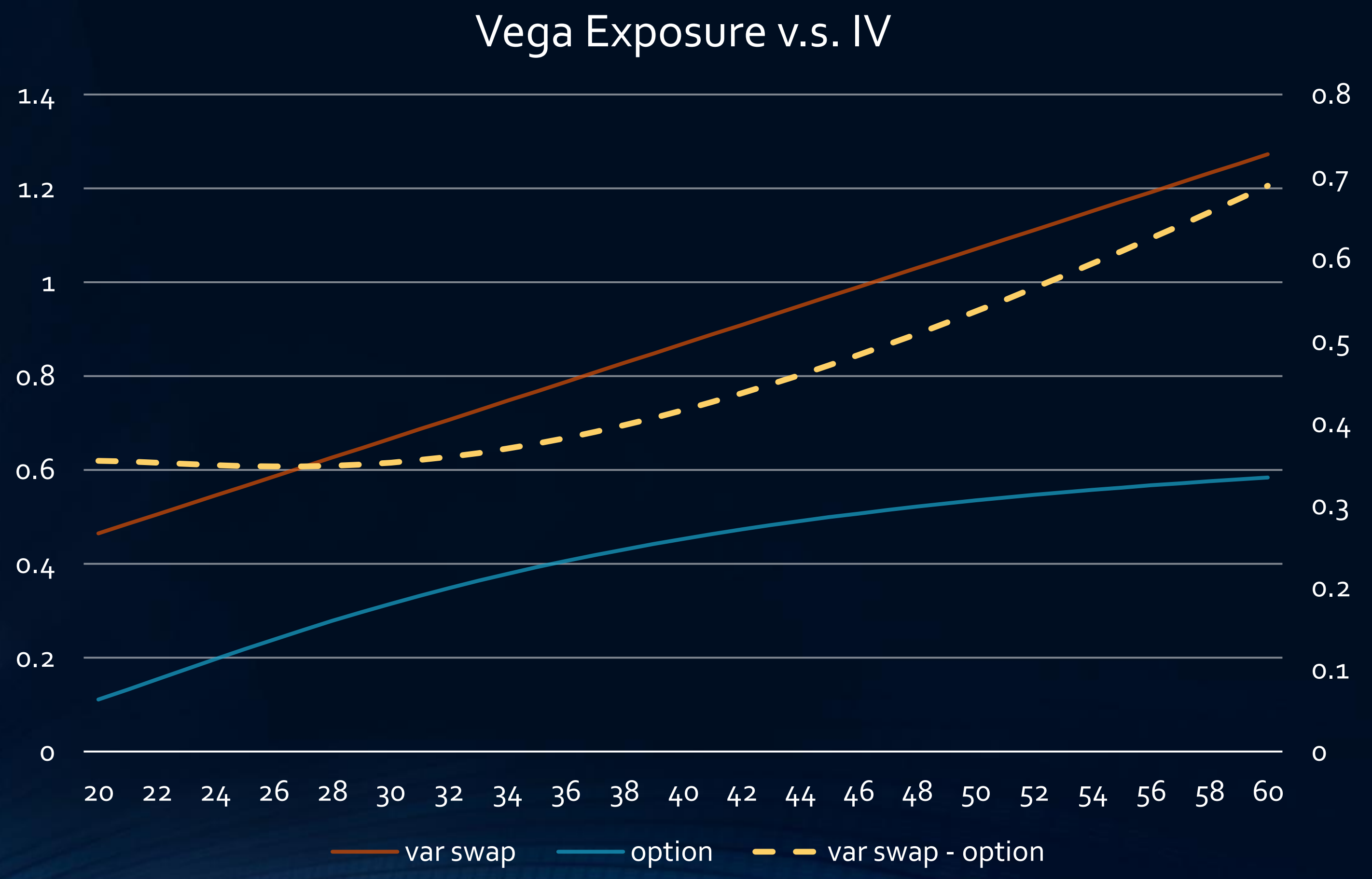


- ❖Long var swap

  Short option:

Example Strategy

- ❖Vega exposure of a 6-month position,
- ❖Assume spot down 20% after 2 months



Example Strategy

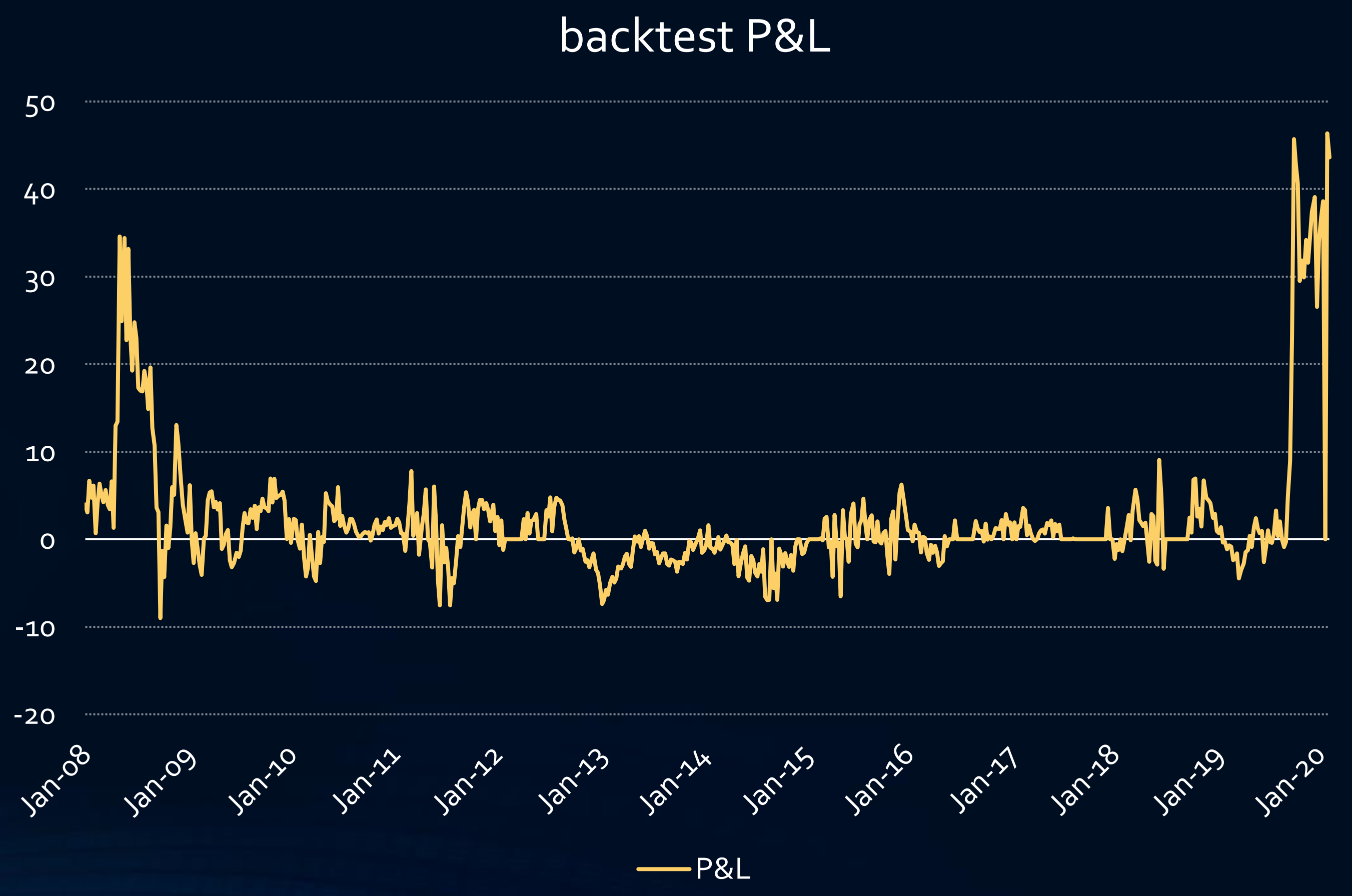
- ❖ Long 1 unit Vega of 6m SPX variance swap,
- ❖ Short 2 unit Vega of 6m SPX atm option, daily hedged, with suitable skew-delta hedging
- ❖ Rolling of strike such that we ensure the strike is always above 95% of the spot
- ❖ Short-dated puts / VIX calls as hedges against moderate declines (-5% ~ -15%)

- ❖ Trade only when $\frac{\sigma_{var}}{\sigma_{atm}} < 1.25$

Example Strategy

Breakdown of average P&L (in vega unit, with 25 bps transaction cost):

- Quiet market (< 5% drawdown): ≈ +0.3
- Mild Correction (5%-10% drawdown): ≈ -0.3
- Medium Correction (10% – 20% drawdown): ≈ +1
- Crash (> 20% drawdown): ≈ 20+



Remarks

- No over-optimization with respect to the recent past.
- This approach lends it self readily to changes in hedging preferences.
- Amenable to further enhancements: e.g. 1) shot less fixed strike vols if vols are oversold; 2) dispense with skew delta hedges if the market is overbought.
- Extendable to non-US markets. For Asian markets, spot-up-vol-up dynamics are far more common. Hence rolling of the strikes needs to adjusted accordingly.

BACH Option

Hong Kong & Connecticut